Homework assignment 4

Solve the following problems without electronic aid.

a) Let V be the real vector space $\mathbb{R}^{3\times 3}$. Provide a subspace of V with a dimension of 6 and justify why your answer is correct.

b) Let W be spanned by the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix} \quad \text{and} \quad \mathbf{v}_3 = \begin{bmatrix} 7 \\ 11 \\ 6 \end{bmatrix}.$$

Provide a basis for W.

c) Let C_∞ be the real vector space from Example 9.34 in the Lecture Notes. Given the function $L : C_\infty \to C_\infty$ defined by $L(f) = f' + f + 1$, determine whether L is a linear map between real vector spaces. As usual the expression f' denotes the derivative function of f.

d) Let $L : \mathbb{C}^3 \to \mathbb{C}^2$ be defined as follows:

$$L\left(\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -4 & 5 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}, \quad v_1, v_2, v_3 \in \mathbb{C}.$$

The following two ordered bases are given:

$$\beta = \left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix}\right) \quad \text{for } \mathbb{C}^2 \text{ and} \quad \gamma = \left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) \quad \text{for } \mathbb{C}^3.$$

Compute the mapping matrix ${}_\beta[L]_\gamma$.

e) Let V denote the real vector space $\mathbb{R}^{2\times 2}$ consisting of 2×2 matrices with real coefficients. The following ordered basis for V is chosen:

$$\beta = \left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right).$$

The linear map $M : V \to V$ is given by

$$M(\mathbf{A}) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \mathbf{A}, \quad \mathbf{A} \in V.$$

Compute the mapping matrix ${}_\beta[M]_\beta$.

f) The following matrix is given:

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix} \in \mathbb{R}^{3\times 3}.$$

Determine the eigenvalues of the matrix and the bases of the corresponding eigenspaces. Can the matrix be diagonalized? If no, explain why not. If yes, provide a matrix $\mathbf{Q}$ and a diagonal matrix $\mathbf{\Lambda}$ such that $\mathbf{Q} \cdot \mathbf{A} \cdot \mathbf{Q}^{-1} = \mathbf{\Lambda}$.

The assignment is to be delivered as a .pdf file by upload on the course's **DTU Learn** module (under "Assignments"). Deadline is **Sunday November 26 at 23:55**.