Homework Assignment 2

Solve the following problems without electronic aid.

a) We are given the complex numbers z_1 and z_2. We are informed that $\mathrm{Arg}(z_1) = \pi/3$ and $\mathrm{Arg}(z_2) = \pi/6$. Show that $z_1 z_2$ is a purely imaginary number.

b) Determine two different solutions to the equation $\mathrm{e}^z = 1 + i$.

c) 1. Show using the division algorithm that the polynomial $Z^2 + 3Z + 2$ is a divisor of the polynomial $Z^4 + 3Z^3 + 4Z^2 + 6Z + 4$ with no remainder.

   2. Now determine all roots of the polynomial $Z^4 + 3Z^3 + 4Z^2 + 6Z + 4$.

d) 1. Show that the number 2 is a root in the polynomial $Z^3 - 3Z^2 + 4$.

   2. Compute the multiplicity of the root 2 in the polynomial $Z^3 - 3Z^2 + 4$.

e) A function $f : \mathbb{N} \to \mathbb{N}$ fulfills that $f(1) = 1$ as well as the following:

$$f(n) = \begin{cases} 2 \cdot f(n/2) & \text{if } n \text{ is even} \\ f(n-1) & \text{if } n \text{ is odd and } n > 1. \end{cases}$$

   Compute $f(1000)$.

f) Given a real number $h \in \mathbb{R}_{>0}$, show using induction on n that

$$(1 + h)^n \geq 1 + nh$$

   for all $n \in \mathbb{Z}_{\geq 0}$.

Your solution must be uploaded as a pdf file to the course's **DTU Learn** module under "Assignments". The deadline is **Sunday October 15 at 23:55**.