Homework set 1: The Essay Exercise

a) Determine whether the following two logical propositions are logically equivalent:

$$(P \Leftrightarrow Q) \vee P \quad \text{and} \quad Q \Rightarrow P.$$

b) State all real numbers x that fulfill the equation $3 \cdot |x| = x^2 + x - 2$.

c) A function $f : \mathbb{R} \to \mathbb{R}$ is given by the expression

$$f(x) = x^2 - x - 3 \cdot |x|.$$

1. Determine whether the function is injective.
2. Compute the image set of the function.

d) We are given the following two subsets of the complex numbers:

$$A = \{z \in \mathbb{C} \mid |z - 1| = 1\} \quad \text{and} \quad B = \{z \in \mathbb{C} \mid \mathrm{Re}(z) = 1\}.$$

1. Draw the sets A and B in the complex plane.
2. Compute $A \cap B$.

e) Show that the complex number $(1 + i)^{300}$ is a real number.

f) As usual the principal argument of a complex number z is denoted by $\mathrm{Arg}(z)$. Determine whether the following propositions are true:

1. $\mathrm{Arg}(z) = 0 \Rightarrow z \in \mathbb{R}$.
2. $z \in \mathbb{R} \Rightarrow \mathrm{Arg}(z) = 0$.

The essay must be uploaded to the **DTU Learn** module for the course via "Assignments". Deadline is **Sunday 24 september at 23:55**.