Homework Assignment 3

Solve the following problems without electronic aid.

a) Compute the rank of the following matrix:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ 2 & 1 & 2 & 1 \\ 0 & 2 & 2 & 1 \end{bmatrix}.$$

b) We are given a certain inhomogeneous system of equations over $\mathbb{R}$ with four equations and three unknowns. We are being informed that the vector

$$\mathbf{v}_p = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \in \mathbb{R}^3$$

is a particular solution to the system. Is the vector $2 \cdot \mathbf{v}_p$ a solution to the system?

c) Determine whether the following four vectors in $\mathbb{R}^3$ are linearly independent:

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ -2 \\ -2 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \quad \text{and} \quad \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}.$$

d) A matrix $\mathbf{B}$ is called symmetric if $\mathbf{B}^T = \mathbf{B}$.

1. Now let $\mathbf{A}$ be an arbitrary $n \times m$ matrix. Show that the matrix $\mathbf{A} \cdot \mathbf{A}^T$ is symmetric.
2. Assume that $\mathbf{B}$ is symmetric and invertible. Is $\mathbf{B}^{-1}$ also symmetric? Justify your answer.

e) Compute the determinant of the following matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 4 & 5 \\ 1 & 1 & 6 & 7 \end{bmatrix}.$$

f) Let n be a natural number and $\mathbf{A}$ as well as $\mathbf{B}$ two $n \times n$ matrices. Assume that both $\mathbf{A}$ and $\mathbf{B}$ are invertible matrices. Show that in that case $(\mathbf{A} \cdot \mathbf{B})^{-1} = \mathbf{B}^{-1} \cdot \mathbf{A}^{-1}$.

The solved problems are to be delivered as a pdf file to the course's **DTU Learn module** under "Assignments". The deadline is **Sunday November 5th, 23:55**.